AFOSR 66-2408

USCAE 102
September 1966

UNIVERSITY OF SOUTHERN CALIFORNIA

SCHOOL OF ENGINEERING

ON THE NEWTONIAN HYPERSONIC STRONG-INTERACTION THEORY
FOR FLOW PAST A FLAT PLATE

William B. Bush

United States Air Force Office of Aerospace Research
Grant No. AF-AFOSR-697-66

DEPARTMENT OF AEROSPACE ENGINEERING

USC Engineering

ON THE NEWTONIAN HYPERSONIC STRONG-INTERACTION THEORY FOR FLOW PAST A FLAT PLATE

William B. Bush*

University of Southern California

Los Angeles, California

Abstract

The viscous hypersonic flow past the leading edge of a sharp flat plate, whose surface is parallel to an oncoming uniform flow, is analysed on the basis of a continuum model consisting of the Navier-Stokes equations and the velocity-slip and temperature-jump wall boundary conditions. It is assumed that the model fluid is a perfect gas having constant specific heats, a constant Prandtl number, σ, whose numerical value is order unity, and a normal viscosity coefficient varying as a power, ω, of the absolute temperature. Limiting forms of the solutions for such a flow are studied as: (1) the free-stream Mach number, M, goes to infinity; (2) the free-stream Reynolds number based upon the distance from the leading edge, R_L, goes to infinity; and (3) the 'Newtonian parameter', $\epsilon=(\gamma-1)/(\gamma+1)$, where γ is the ratio of the specific heats, goes to zero; such that the Newtonian hypersonic interaction parameter, $\chi_\epsilon=(\epsilon^{3+\omega}M^{2(2+\omega)}/R_L)^{1/2}$, goes to infinity.

Through the use of asymptotic expansions and matching, it is shown that, for $\omega<1$, near the leading edge, the interaction of the thin (thickness ratio, δ, going to zero), viscous, high temperature, principal layer adjacent to the plate and the external flow produces three distinct strong-interaction ($N=1/\epsilon M^2\delta^2 \ll 1$)(sub-)regimes, for which the quantity δ is of order $\epsilon^m (m>1/(1+\omega))$, $\epsilon^{1/(1+\omega)}$, and $\epsilon^{1/2}$, respectively, as the various limits are approached.

1. Introduction

According to ordinary hypersonic strong-interaction theory

This work was partially supported by the U. S. Air Force under Grant No. AF-AFOSR-697-66.

*Assistant Professor, Department of Aerospace Engineering.

(OHSIT) with $\epsilon = O(1)$, for flow past a flat plate (Bush,[1]), for $\omega<1$, at the plate surface, there is a thin, viscous, heat conducting layer, which disturbs the external uniform, high speed flow. This layer, with δ, the layer's thickness ratio, much less than $O(1)$, acts as an effective slender 'body', producing an oblique Rankine-Hugoniot shock wave (SW), whose distance from the plate is also of $O(\delta)$, and an HSDT inviscid shock layer (SL) between the clearly defined outer edge of the viscous 'body' and the downstream side of the SW. In addition, it is found that there must be a viscous HSDT-type transition layer, whose thickness ratio is much less than $O(\delta)$, between the viscous 'body' and the inviscid SL, in order to insure uniform matching between these two layers. This theory is shown to be valid for $\chi = M^{2+\omega}/R_L^{1/2} \gg 1$.

The Newtonian hypersonic strong-interaction theory (NHSIT) with $\epsilon \ll 1$, is formulated, assuming that the Newtonian equivalent of the above requirement holds true, namely, $\chi_\epsilon = \epsilon^{(3+\omega)/2} M^{2+\omega}/R_L^{1/2} \gg 1$.

Clearly, it is expected that the NHSIT should produce a regime that is the direct equivalent of the OHSIT picture just described. However, with the degree of freedom that the additional limit of $\epsilon \to 0$ offers, it is also expected that the NHSIT should produce additional (sub-)regimes.

In the following sections, it is shown that, indeed, the NHSIT does produce the direct equivalent of the OHSIT regime plus two additional (sub-)regimes that owe their existence entirely to ϵ's approaching zero.

2. The Equations of Motion

Consider the (two-dimensional) flow of a viscous, compressible gas past a semi-infinite flat plate. Let $x_1 = Lx$ and $y_1 = Ly$ represent the Cartesian coordinates parallel and normal to the plate, respectively, with the origin of this coordinate system at the leading edge of the plate. The length L is to be chosen so that x is of $O(1)$ in each regime that is analysed. The velocity components in the x_1- and y_1-directions are $u_1 = u_\infty u$ and $v_1 = u_\infty v$, and the pressure, temperature, and density are $p_1 = p_\infty p$, $T_1 = T_\infty T$, and $\rho_1 = \rho_\infty \rho$, where u_∞, p_∞, T_∞, and ρ_∞ are the velocity in the x_1-direction, pressure, temperature, and density in the undisturbed region upstream of the flat plate.

The gas is assumed to be a perfect one ($p = \rho T$), having (i) constant specific heats, c_{v_1} and c_{p_1}, with $\gamma = (c_{p_1}/c_{v_1})$, such that $\epsilon = (\gamma-1)/(\gamma+1) \ll 1$; (ii) a constant Prandtl number of order unity ($\sigma = \text{const.} = O(1)$); and (iii) a normal viscosity coefficient proportional to a power, ω, of the absolute temperature ($\mu_1 = \mu_\infty \mu = \mu_\infty T^\omega$,

with $1/2 \leq \omega < 1$) and a bulk viscosity coefficient taken to be zero.

The two-dimensional Navier-Stokes equations including the Fourier heat conduction law, assumed to be the governing equations of motion for the flow of such a gas, are

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0\,, \tag{2.01}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{1-\epsilon}{1+\epsilon}\,\frac{1}{M^2}\frac{\partial p}{\partial x} = \frac{1}{R_L}\left[\frac{\partial}{\partial y}\left(T^{\omega}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}\right) + \frac{\partial}{\partial x}\left(T^{\omega}\left\{\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y}\right\}\right)\right], \tag{2.02}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \frac{1-\epsilon}{1+\epsilon}\,\frac{1}{M^2}\frac{\partial p}{\partial y} = \frac{1}{R_L}\left[\frac{\partial}{\partial y}\left(T^{\omega}\left\{\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x}\right\}\right) + \frac{\partial}{\partial x}\left(T^{\omega}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}\right)\right], \tag{2.03}$$

$$\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \frac{2\epsilon}{1+\epsilon}\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) = \frac{1}{\sigma R_L}\left[\frac{\partial}{\partial y}\left(T^{\omega}\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial x}\left(T^{\omega}\frac{\partial T}{\partial x}\right)\right]$$

$$+ \frac{2\epsilon M^2}{(1-\epsilon)R_L}\,T^{\omega}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left(\left\{\frac{\partial u}{\partial x}\right\}^2 + \left\{\frac{\partial v}{\partial y}\right\}^2\right) - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right], \tag{2.04}$$

where $M^2 = (\rho_\infty u_\infty^2/\gamma p_\infty)$, the square of the free-stream Mach number, and $R_L = (\rho_\infty u_\infty L/\mu_\infty)$, the Reynolds number. The analysis presented here is for $M^2 >> 1$ and $R_L >> 1$.

3. The Principal Layer

According to existing hypersonic strong-interaction theory for flow past a flat plate, at the surface there is a thin, viscous, heat conducting layer, which disturbs the external flow. This layer, designated here as the principal layer (PL), whose outer edge is given by $y = \delta Y_b(x)$, with δ, the layer's thickness ratio, much less than $O(1)$, has as its proper distorted Cartesian coordinates

$$x = x_b, \quad y = \delta y_b\,. \tag{3.01}$$

The expansions for the flow quantities in this region, carried out in the above distorted coordinates, are taken to have the form

$$u = u_b + \cdots,\ v = \delta v_b + \cdots,\ p = (M^2\delta^2)p_b + \cdots,\ T = (\epsilon M^2)T_b + \cdots,\ \rho = (\delta^2/\epsilon)\rho_b + \cdots, \tag{3.02}$$

with the variables f_b of $O(1)$.

For these expansions, in which it is implicit that

$$N = 1/\epsilon M^2\delta^2 << 1, \tag{3.03}$$

the leading terms in the equations of motion are

$$\frac{\partial}{\partial x_b}(\rho_b u_b)+\frac{\partial}{\partial y_b}(\rho_b v_b)=0,\; p_b=\rho_b T_b,\; \rho_b\left(u_b\frac{\partial u_b}{\partial x_b}+v_b\frac{\partial u_b}{\partial y_b}\right)=\lambda\left[\frac{\partial}{\partial y_b}\left(T_b^{\omega}\frac{\partial u_b}{\partial y_b}\right)\right],$$

$$\frac{\partial p_b}{\partial y_b}+\frac{1}{K_b}\rho_b\left(u_b\frac{\partial v_b}{\partial x_b}+v_b\frac{\partial v_b}{\partial y_b}\right)=\frac{\lambda}{K_b}\left[\frac{\partial}{\partial y_b}\left(T_b^{\omega}\left\{\frac{4}{3}\frac{\partial v_b}{\partial y_b}-\frac{2}{3}\frac{\partial u_b}{\partial x_b}\right\}\right)+\frac{\partial}{\partial x_b}\left(T_b^{\omega}\frac{\partial u_b}{\partial y_b}\right)\right],$$

$$\rho_b\left(u_b\frac{\partial T_b}{\partial x_b}+v_b\frac{\partial T_b}{\partial y_b}\right)=\lambda\left[\frac{1}{\sigma}\frac{\partial}{\partial y_b}\left(T_b^{\omega}\frac{\partial T_b}{\partial y_b}\right)+2\,T_b^{\omega}\left(\frac{\partial u_b}{\partial y_b}\right)^2\right], \qquad (3.04)$$

where $\lambda=\epsilon^{1+\omega}M^{2\omega}/R_L\delta^4$ and $K_b=(\epsilon/\delta^2)$. The quantity $\lambda=O(1)$, in order that there be a balance between the inviscid and viscous terms, while K_b must be $\geqslant O(1)$($K_b<<1$ is rules out as not being physically realistic for this flow problem). Note: The combination of $\lambda=O(1)$ and $N<<1$ yields

$$\chi_\epsilon=\epsilon^{(3+\omega)/2}M^{2+\omega}/R_L^{1/2}>>1.$$

From Eq.(3.04), it is seen that there are two different structures for this layer depending upon the magnitude of the parameter K_b. For $K_b>>1$, the y-momentum equation becomes

$$\frac{\partial p_b}{\partial y_b}=0,$$

and Eq.(3.04) corresponds exactly to the flat plate boundary layer equations. For $K_b=O(1)$, although the rest of the equations are of flat plate boundary layer type, the y-momentum equation is not, since, in this limit, the convection and viscosity terms, as well as the normal pressure gradient term, are retained. The PL with $K_b>>1$ is the Newtonian equivalent of the OHSIT viscous boundary layer, while the PL with $K_b=O(1)$ is a special case, for which there is no OHSIT parallel.

To complete the picture, the boundary conditions that are to be applied to the PL's equations of motion must be specified.

The boundary conditions at the wall ($y_1=0$), taking into account velocity-slip, thermal-creep, and temperature-jump, are

$$u_{1,0}\doteq C_1\left\{\mu_1(p_1\rho_1)^{-1/2}\frac{\partial u_1}{\partial y_1}\right\}_{,0}+\frac{3}{4}\left\{\mu_1(\rho_1 T_1)^{-1}\frac{\partial T_1}{\partial x_1}\right\}_{,0},$$

$$v_{1,0}=0,\; T_{1,0}-T_{1,w}=C_2\left\{\mu_1(p_1\rho_1)^{-1/2}\frac{\partial T_1}{\partial y_1}\right\}_{,0}, \qquad (3.05)$$

where $T_{1,w}$ is the temperature of the plate, and C_1 and C_2 are

constants of $O(1)$, which depend upon the momentum and thermal accommodation coefficients of the surface and the gas properties. Recasting Eq.(3.05) in terms of the PL variables gives, at $y_b = 0$,

$$u_{b,o} - \frac{\lambda C_1}{K_b^{1/2}} \left\{ \frac{T_b^{(2\omega-1)/2}}{\rho_b} \frac{\partial u_b}{\partial y_b} \right\}_{,o} = 0, v_{b,o} = 0,$$

$$T_{b,o} - T_{b,w} - \frac{\lambda C_2}{K_b^{1/2}} \left\{ \frac{T_b^{(2\omega-1)/2}}{\rho_b} \frac{\partial T_b}{\partial y_b} \right\}_{,o} = 0, \qquad (3.06)$$

with $T_{b,w} = T_{1,w}/\epsilon M^2 T_\infty$. Again, two different structures arise depending on the parameter K_b. For $K_b \gg 1$, the wall boundary conditions reduce to

$$u_{b,o} = v_{b,o} = T_{b,o} - T_{b,w} = 0 \,,$$

the usual non-slip boundary conditions. On the other hand, for $K_b = O(1)$, the velocity-slip and temperature-jump effects must be retained. Even for $K_b = O(1)$, the effect of thermal-creep is negligible.

Thus, for $K_b >> 1$, which corresponds to $\delta << \epsilon^{1/2}$, the equations of the PL take the form of the flat plate boundary layer equations and satisfy the non-slip boundary conditions at the wall; while, for $K_b = O(1) (\delta = O(\epsilon^{1/2}))$, the PL equations are of flat plate boundary layer type except for the y-momentum equation, and satisfy the slip boundary conditions at the wall. These two couplings of the equations and wall boundary conditions for the PL are the only consistent couplings (cf., Pan and Probstein, [2], [3] ; Garvine, [4], [5]).

The specification of the boundary conditions at the outer edge of the PL cannot be made explicitly at this time, since these conditions depend upon matching with the solutions for the adjacent layer.

4. The Equations of Motion for the External Layers

The analysis of the flow in the layers external to the PL is most conveniently done, assuming the PL to be thicker than any of these external layers, in terms of a boundary layer type coordinate system oriented with respect to the outer edge of the PL.

If $s_1 = Ls$ and $n_1 = Ln$ are the boundary layer type coordinates, with the s_1-axis along the PL's outer edge and the n_1-axis normal to this outer edge, then the Cartesian and boundary layer coordinate systems are related by

$$x = \int_0^s \cos\Phi(t)dt - n\sin\Phi(s), \quad y = \int_0^s \sin\Phi(t)dt + n\cos\Phi(s), \qquad (4.01)$$

where $\Phi(s)$ is the angle that the PL's outer edge makes with the free-stream direction. The velocity components in the s_1-and n_1-directions are $q_{s,1} = u_\infty q_s$ and $q_{n,1} = u_\infty q_n$. The velocity components of these two coordinate systems are related by

$$u = q_s\cos\Phi - q_n\sin\Phi, \quad v = q_s\sin\Phi + q_n\cos\Phi. \qquad (4.02)$$

In this new coordinate system, Eqs.(2.01)-(2.04) become

$$\frac{\partial}{\partial s}(\rho q_s) + \frac{\partial}{\partial n}(h\rho q_n) = 0, \quad \rho\left(\frac{q_s}{h}\frac{\partial q_s}{\partial s} + q_n\frac{\partial q_s}{\partial n} - \frac{\Phi'}{h}q_s q_n\right) + \frac{1-\epsilon}{1+\epsilon}\frac{1}{M^2}\frac{1}{h}\frac{\partial p}{\partial s}$$

$$= \frac{1}{R_L}\left[\left(\frac{\partial}{\partial n} - \frac{2\Phi'}{h}\right)\left(T^\omega\left\{\left(\frac{\partial}{\partial n} + \frac{\Phi'}{h}\right)q_s + \frac{1}{h}\frac{\partial q_n}{\partial s}\right\}\right)\right.$$

$$\left. + \frac{1}{h}\frac{\partial}{\partial s}\left(T^\omega\left\{\frac{4}{3}\frac{1}{h}\frac{\partial q_s}{\partial s} - \frac{2}{3}\left(\frac{\partial}{\partial n} + \frac{2\Phi'}{h}\right)q_n\right\}\right)\right],$$

$$\rho\left(\frac{q_s}{h}\frac{\partial q_n}{\partial s} + q_n\frac{\partial q_n}{\partial n} + \frac{\Phi'}{h}q_s^2\right) + \frac{1-\epsilon}{1+\epsilon}\frac{1}{M^2}\frac{\partial p}{\partial n} = \frac{1}{R_L}\left[\left(\frac{4}{3}\frac{\partial}{\partial n} - \frac{2\Phi'}{h}\right)\left(T^\omega\frac{\partial q_n}{\partial n}\right)\right.$$

$$+ \frac{1}{h}\frac{\partial}{\partial s}\left(T^\omega\left\{\left(\frac{\partial}{\partial n} + \frac{\Phi'}{h}\right)q_s + \frac{1}{h}\frac{\partial q_n}{\partial s}\right\}\right)$$

$$\left. - \left(\frac{2}{3}\frac{\partial}{\partial n} - \frac{2\Phi'}{h}\right)\left(T^\omega\left\{\frac{1}{h}\left(\frac{\partial q_s}{\partial s} - \Phi' q_n\right)\right\}\right)\right],$$

$$\rho\left(\frac{q_s}{h}\frac{\partial T}{\partial s} + q_n\frac{\partial T}{\partial n}\right) - \frac{2\epsilon}{1+\epsilon}\left(\frac{q_s}{h}\frac{\partial p}{\partial s} + q_n\frac{\partial p}{\partial n}\right)$$

$$= \frac{1}{\sigma R_L}\left[\left(\frac{\partial}{\partial n} - \frac{\Phi'}{h}\right)\left(T^\omega\frac{\partial T}{\partial n}\right) + \frac{1}{h}\frac{\partial}{\partial s}\left(T^\omega\left\{\frac{1}{h}\frac{\partial T}{\partial s}\right\}\right)\right]$$

$$+ \frac{2\epsilon M^2}{(1-\epsilon)R_L}T^\omega\left[2\left(\frac{\partial q_n}{\partial n}\right)^2 + 2\left(\frac{1}{h}\left\{\frac{\partial q_s}{\partial s} - \Phi' q_n\right\}\right)^2\right.$$

$$\left. + \left(\left(\frac{\partial}{\partial n} + \frac{\Phi'}{h}\right)q_s + \frac{1}{h}\frac{\partial q_n}{\partial s}\right)^2 - \frac{2}{3}\left(\frac{1}{h}\frac{\partial q_s}{\partial s} + \left(\frac{\partial}{\partial n} - \frac{\Phi'}{h}\right)q_n\right)^2\right], \qquad (4.03)$$

where $h = 1 - \Phi' n$.

Under the assumptions that the PL has a thickness ratio of $\delta << 1$, and the thickness ratio of any external layer is $<< O(\delta)$, it follows that

$$s \to x, \ \Phi \to \delta\phi \ (\sin\Phi \to \delta\phi, \ \cos\Phi \to 1, \ h \to 1), \tag{4.04a}$$

where, since the outer edge of the PL is $y_{b,E} = Y_b(x_b)$,

$$\phi \to Y_b'. \tag{4.04b}$$

5. The Shock Layer

The PL, as formulated in Sec.3, with $K_b >> 1$, acting as an effective slender 'body', produces an oblique SW and a SL between the 'body' and the SW. The structure of the Newtonian SL is given in this section, using the Newtonian HSDT (Cole, [6]) as a guide for determining the orders of magnitude of the flow quantities. According to this theory, valid only for $K_b >> 1$, the coordinates of the layer are

$$s = s_h, \ n = (\epsilon\delta)n_h \tag{5.01}$$

and the flow quantities are expressible as

$$q_s = 1 + \delta^2 u_h + \cdots, \ q_n = (\epsilon\delta)v_h + \cdots,$$

$$p = (M^2\delta^2)p_h + \cdots, \ T = (\epsilon M^2\delta^2)T_h + \cdots, \ \rho = (1/\epsilon)\rho_h + \cdots, \tag{5.02}$$

with the variables f_h of $O(1)$.

For this layer, then, the leading terms in the equations of motion, Eq. (4.03), are

$$\frac{\partial\rho_h}{\partial s_h} + \frac{\partial}{\partial n_h}(\rho_h v_h) = 0, \ p_h = \rho_h T_h, \ \rho_h\phi' + \frac{\partial p_h}{\partial n_h} = 0,$$

$$\rho_h\left(\frac{\partial u_h}{\partial s_h} + v_h\frac{\partial u_h}{\partial n_h}\right) = \frac{\lambda}{K_h^2}\left[\frac{\partial}{\partial n_h}\left(T_h^\omega\frac{\partial u_h}{\partial n_h}\right)\right], \rho_h\left(\frac{\partial T_h}{\partial s_h} + v_h\frac{\partial T_h}{\partial n_h}\right) = \frac{\lambda}{K_h^2}\left[\frac{1}{\sigma}\frac{\partial}{\partial n_h}\left(T_h^\omega\frac{\partial T_h}{\partial n_h}\right)\right], \tag{5.03}$$

where $K_h = (\epsilon/\delta^{1+\omega})$.

Since $\lambda = O(1)$, the ratios of the orders of magnitude of the leading viscosity and heat conduction terms to those of the inviscid terms in Eq.(5.03) are $O(1/K_h^2)$. Thus, within the Newtonian framework, Eq.(5.03) describes either an inviscid

shock layer (ISL) or a viscous shock layer (VSL), depending on whether the quantity K_h is greater than or equal to $O(1)$. The ISL, with $K_h \gg 1$, is the Newtonian equivalent of the OHSIT SL, while the VSL, with $K_h = O(1)$, is a product of the Newtonian approximation that $\epsilon \to 0$, and has no CHSIT parallel. (Note:

$$K_h/K_b = \delta^{1-\omega} \ll 1 \text{ for } \omega < 1,$$

and the relation $K_h \geq O(1)$ does not violate the condition $K_b \gg 1$.)

To complete the specification of the SL, it is necessary to determine the shock relations that are to be applied at the outer edge of the layer, $n_{h,E} = N_h(s_h)$, for $N \ll 1$, and show that the resulting solutions match at the SL inner edge to the PL solutions.

6. The Shock Structure and Shock Relations

An analysis, similar to the one performed to obtain the shock structure for viscous hypersonic blunt body theory (Bush, [7]), finds that either PL-SL combination, with $N \ll 1$, and $K_b \gg 1$, supports a shock structure (SS) which is made up of three thin layers: (1) a very thin exterior layer, in which the order of magnitude of the flow quantities is characterized by their magnitude in the free-stream; (2) a relatively thicker middle layer, in which there is dissipation; and (3) a thin interior layer, which acts as a transition layer between the middle layer of the SS and the SL.

Since, as has been noted, the analysis of the three layers of the SS for NHSIT follows the lines of that given in [7], only the proper expansions of the flow quantities and the limiting forms of the equations of motion for these layers are presented.

6.1. The Uniform Upstream Region

In the uniform upstream region the flow quantities are

$$q_s = \cos\Phi = 1 - \delta^2(\phi^2/2) + \cdots, \quad q_n = -\sin\Phi = -\delta\phi + \cdots, \quad p = \rho = T = 1\,. \qquad (6.01)$$

6.2. The Exterior Layer of the Shock Structure

The coordinates of the exterior layer (EL) are

$$s = s_e,\ n = \delta\left[\epsilon Z_e(s_e) + \left(\frac{1}{K_b(\epsilon M^2)^{\omega}}\right)n_e\right] = \epsilon\delta\left[Z_e(s_e) + \left(\frac{N^{\omega}}{K_h^2}\right)n_e\right] = \epsilon\delta\left[N_h(s_h) + \left(\frac{N^{\omega}}{K_h^2}\right)n_e + \cdots\right], \qquad (6.02)$$

and the leading terms in the expansions for the flow quantities are

$$q_s = \cos\Phi + \delta^2 N^{1/\sigma} u_e + \cdots = \{1 - \delta^2(\phi^2/2) + \cdots\} + \delta^2 N^{1/\sigma} u_e + \cdots,$$

$$q_n = -\sin\Phi + \delta N^{3/4\,\sigma} v_e + \cdots = \{-\delta\phi + \cdots\} + \delta N^{3/4\,\sigma} v_e + \cdots,$$

$$\rho = 1 + N^{3/4\sigma}\rho_e + \cdots, \quad p = p_e + \cdots, \quad T = T_e + \cdots \quad . \tag{6.03}$$

The first integrals of the equations of motion for this layer, that match to the upstream boundary conditions of Eq.(6.01), are

$$v_e - \phi\rho_e = 0, \; p_e - T_e = 0, \; \phi u_e + \lambda\left[T_e^{\omega}\left(\frac{\partial u_e}{\partial n_e} - \left\{\frac{\epsilon}{N^{1/4\,\sigma}}\right\}\frac{Z'_e}{3}\frac{\partial v_e}{\partial n_e}\right)\right] = 0,$$

$$\phi\, v_e + \lambda\left[\frac{4}{3}T_e^{\omega}\frac{\partial v_e}{\partial n_e}\right] = 0, \; \phi(T_e - 1) + \lambda\left[\frac{1}{\sigma}T_e^{\omega}\frac{\partial T_e}{\partial n_e}\right] = 0, \tag{6.04}$$

subject to the restriction of $N \geqslant O(\epsilon^{4\sigma})$. This restriction must be consistent with the restriction from Sec.5 of $N \geqslant O(1/M^2\epsilon^{(\beta+\omega)/(1+\omega)})$.

6.3. The Middle Layer of the Shock Structure

The coordinates of the middle layer (ML) are

$$s = s_m, \; n = \epsilon\delta\left[Z_m(s_m) + \left(\frac{1}{K_h^2}\right)n_m\right] = \epsilon\delta\left[N_h(s_h) + \left(\frac{1}{K_h^2}\right)n_m + \cdots\right], \tag{6.05}$$

and the leading terms in the flow quantity expansions are

$$q_s = \cos\Phi + \delta^2 u_m + \cdots = 1 + \delta^2\left(u_m - \frac{\phi^2}{2}\right) + \cdots, \; q_n = -\sin\Phi + \delta v_m + \cdots = \delta(v_m - \phi) + \cdots,$$

$$\rho = 1 + \rho_m + \cdots, \; p = (1/N)p_m + \cdots, \; T = (1/N)T_m + \cdots \quad . \tag{6.06}$$

The first integrals of the equations of motion, determined by matching with the EL, are

$$(1+\rho_m)(v_m - \phi) + \phi = 0, \; p_m - (1+\rho_m)T_m = 0, \; \phi u_m + \lambda\left[T_m^{\omega}\frac{\partial u_m}{\partial n_m}\right] = 0,$$

$$\phi v_m + \lambda\left[\frac{4}{3}T_m^{\omega}\frac{\partial v_m}{\partial n_m}\right] = 0, \; \phi(T_m - v_m^2) + \lambda\left[\frac{1}{\sigma}T_m^{\omega}\frac{\partial T_m}{\partial n_m}\right] = 0 \; . \tag{6.07}$$

6.4. The Inner Layer of the Shock Structure

The quantities in the inner layer (IL) have the following representations:

$$s = s_i, n = \epsilon\delta\left[Z_i(s_i) + \left(\frac{\epsilon}{K_h^2}\right)n_i\right] = \epsilon\delta\left[N_h(s_h) + \left(\frac{\epsilon}{K_h^2}\right)n_i\right]. \qquad (6.08)$$

$$q_s = \cos\Phi + \delta^2\left[W(s_i) + \epsilon\, u_i + \cdots\right] = 1 + \delta^2\{W - (\phi^2/2)\} + \epsilon\delta^2 u_i + \cdots,$$

$$q_n = (\epsilon\delta)v_i + \cdots,\ \rho = (1/\epsilon)\rho_i + \cdots,\ p = (1/\epsilon N)p_i + \cdots,\ T = (1/N)\left[Q(s_i) + \epsilon T_i + \cdots\right], \qquad (6.09)$$

The first integrals of the equations of motion, from the ML-IL matching, are

$$\rho_i(v_i - N_h') + \phi = \rho_i v_i^* + \phi = 0,\ p_i - Q\rho_i = 0,\ \lambda\left[Q^\omega \frac{\partial u_i}{\partial n_i}\right] + W\phi = 0,\ p_i - \lambda\left[\frac{4}{3}Q^\omega \frac{\partial v_i^*}{\partial n_i}\right] - \phi^2 = 0,$$

$$\lambda\left[\frac{1}{\sigma}Q^\omega \frac{\partial T_i}{\partial n_i}\right] + \phi\,(Q - \phi^2) = 0. \qquad (6.10)$$

6.5. The Shock Relations

From matching with the solutions of the IL (as $n_i \to -\infty$), the boundary conditions at the outer edge of the SL, $N_h(s_h)$, are determined to be

$$u_{h,E} + \frac{\lambda}{K_h^2}\frac{1}{\phi}\left(T_h^\omega \frac{\partial u_h}{\partial n_h}\right)_{,E} = -\phi^2/2,\ T_{h,E} + \frac{\lambda}{K_h^2}\frac{1}{\phi}\left(\frac{T_h^\omega}{\sigma}\frac{\partial T_h}{\partial n_h}\right)_{,E} = \phi^2,$$

$$v_{h,E} = N_h' - (T_{h,E}/\phi),\ \rho_{h,E} = \phi^2/T_{h,E},\ p_{h,E} = \phi^2, \qquad (6.11)$$

where $f_{h,E} = f_h(s_h, N_h(s_h))$.

Before discussing the inviscid shock layer regime ($K_h >> 1$) and the viscous shock layer regime ($K_h = O(1)$), it must be re-emphasized that the SS analysis presented above is valid only for $K_b >> 1$. Therefore, for $K_b = O(1)$, the whole concept of a SW, as well as that of a SL, must be modified.

7. The Inviscid Shock Layer Regime: $\delta << \epsilon^{1/(1+\omega)}$

For the inviscid shock layer regime (ISLR), $K_h \gg 1$ ($K_b \gg 1$), the primary layers of interest are the PL (defined by Eqs. (3.04) and (3.06), with $K_b \gg 1$), and the ISL (defined by Eqs. (5.03) and (6.11), with $K_h \gg 1$).

It is possible to write the equations for these layers and the boundary conditions at the SW and at the plate in imilarity

form, as is done for OHSIT in [1], if $N \ll 1$ and $Y_b(x_b) \sim x_b^{3/4}$ and $T_{b,w}$ = const. However, as in the OHSIT case, the solutions of the PL and ISL do not match directly to each other, and a transition layer (TL), intermediate to the PL and ISL, is required to provide uniform matching for the entire ISLR.

Since the discussion of such PL-TL-ISL similarity solutions is presented in detail in [1], in this paper, it is considered sufficient to present the equations of motion for the Newtonian TL, showing the dependence of the flow quantities in this region upon $K_h \gg 1$.

The quantities in the TL have the following representations:

$$s = s_t, \quad n = (\epsilon\,\delta)\Delta_t n_t, \tag{7.01a}$$

$$q_s = 1 + \delta^2 \alpha_t u_t + \cdots, \quad q_n = (\epsilon\,\delta)\beta_t v_t + \cdots,$$

$$p = (M^2\delta^2)p_t + \cdots, \quad T = (\epsilon M^2 \delta^2)\theta_t T_t + \cdots, \quad \rho = (1/\epsilon)(1/\theta_t)\rho_t + \cdots, \tag{7.01b}$$

where

$$\theta_t = \alpha_t = K_h^{2/(2+\omega)} \gg 1, \Delta_t = \beta_t = K_h^{-1/(2+\omega)} \ll 1, \text{for } K_h \gg 1, \tag{7.02}$$

and the variables f_t are of $O(1)$. (Note: It is also true that

$$(\epsilon M^2 \delta^2)\theta_t = (\epsilon M^2)(\epsilon\,\delta)^{2/(2+\omega)} \ll \epsilon M^2, \delta^2 \alpha_t = (\epsilon\,\delta)^{2/(2+\omega)} \ll 1.\ )$$

From Eq.(7.01), then, the leading terms in the equations of motion for this TL are

$$\frac{\partial \rho_t}{\partial s_t} + \frac{\partial}{\partial n_t}(\rho_t v_t) = 0, \quad p_t = \rho_t T_t, \frac{\partial p_t}{\partial n_t} = 0,$$

$$\rho_t\left(\frac{\partial u_t}{\partial s_t} + v_t \frac{\partial u_t}{\partial n_t}\right) = \lambda\left[\frac{\partial}{\partial n_t}\left(T_t^{\omega}\frac{\partial u_t}{\partial n_t}\right)\right], \quad \rho_t\left(\frac{\partial T_t}{\partial s_t} + v_t \frac{\partial T_t}{\partial n_t}\right) = \lambda\left[\frac{1}{\sigma}\frac{\partial}{\partial n_t}\left(T_t^{\omega}\frac{\partial T_t}{\partial n_t}\right)\right] \tag{7.03}$$

8. The Viscous Shock Layer Regime: $\delta = O(\epsilon^{1/(1+\omega)})$

For the viscous shock layer regime (VSLR), $K_h = O(1)$ ($K_b \gg 1$), the primary layers of interest are the PL (defined by Eqs.(3.04) and (3.06), with $K_b \gg 1$), and the VSL (defined by Eqs.(5.03) and (6.11), with $K_h = O(1)$). (Note: For $K_h = O(1)$, the thickness ratio of the ML of the SS is of the same order of

magnitude as that of the SL, namely, $O(\epsilon\delta)$.)

For this regime, it is not possible to write the equations and boundary conditions for these layers in similarity form, due to the additional terms in the shock relations. However, it is still possible to show that, since the VSL equations now contain the viscous terms that are required in the TL equations for the TL's matching with the PL (cf., Eq. (7.03)), the VSL matches directly with the PL, and no intermediate layer is required.

9. The Merged Layer Regime: $\delta = O(\epsilon^{1/2})$

Consider that $K_b = O(1)$, so that $\delta = O(\epsilon^{1/2}) \ll 1$. When this is the case, the analyses presented for the SL and SS cannot be valid (cf., Sec. 5 and 6). However, for $K_b = O(1)$, Eq. (3.04), giving the equations of motion for the PL, contains all the terms that are present in the SL and SS equations, as well as those in the viscous boundary layer equations. Therefore, it is concluded that, for $K_b = O(1)$, a merged layer regime (MLR), upstream of the regimes of Secs. 7 and 8, exists, in which the three above-mentioned regions are replaced by a single PL, which takes on all of their properties.

Strictly speaking, since T and p are of $O(\epsilon M^2) \gg 1$ in the PL of the MLR, this PL now includes only what were formerly the SL and the ML and IL of the SS, and must be complemented by an EL, similar to the one formulated in Sec. 6.2, with $Z_e = 0$, in which T and p are of $O(1)$. From the matching with this EL, the boundary conditions at the outer edge of this PL, $Y_b(x_b)$, are found to be

$$u_{b,E} = 1, v_{b,E} = 0, \rho_{b,E} = K_b, T_{b,E} = p_{b,E} = 0 \left(f_{b,E} = f_b(x_b, Y_b(x_b))\right). \tag{9.01}$$

The boundary conditions at the wall are those given in Eq. (3.06) for $K_b = O(1)$. Further, as $x_b \to 0$, the flow quantities at the wall should also approach their free-stream values, namely,

$$u_{b,00} = 1, v_{b,00} = 0, \rho_{b,00} = K_b, T_{b,00} = p_{b,00} = 0 \left(f_{b,00} = f_b(0, 0)\right). \tag{9.02}$$

An examination of Eqs. (3.04), (3.06), (9.01), and (9.02) shows that, for this regime, it is not possible to write this set of equations in a general similarity form, however, it is possible to show that local similarity can exist near $x_b = 0$.

Consider that, as $x_b \to 0$, the coordinates and flow quantities may be expressed as

$$x_b = \xi_b, \; y_b = Y_b(\xi_b)\eta_b, \text{with } Y_b(\xi_b) = A^*\xi_b^{J_y} + \cdots \; (A^* = \text{const.}); \qquad (9.03)$$

$$u_b = 1 + \xi_b^{J_u} U_b^*(\eta_b) + \cdots, \; v_b = \xi_b^{J_v} V_b^*(\eta_b) + \cdots, \; \rho_b = K_b + \xi_b^{J_\rho} R_b^*(\eta_b) + \cdots,$$

$$T_b = \xi_b^{J_T} H_b^*(\eta_b) + \cdots, \; p_b = K_b \xi_b^{J_T} H_b^*(\eta_b) + \cdots, \qquad (9.04)$$

with J_y, J_u, J_v, J_ρ, $J_T > 0$.

Take the values of these exponents to be

$$1/2 < J_u = 2J_v = J_\rho = J_T = 1/(1+\omega) \leq 2/3,$$
$$2/3 \leq J_y = (2\omega+1)/2(1+\omega) < 3/4 \; (\text{since } 1/2 \leq \omega < 1). \qquad (9.05)$$

(Note: The value taken for J_y in Eq.(9.05) indicates that the effective 'body' becomes progressively blunter as the leading edge is approached, rather than becoming straight.) Substitution of the above expressions into Eq.(3.04) (with $K_b = O(1)$) yields

$$\left[J_T U_b^* - J_y \eta_b \frac{dU_b^*}{d\eta_b}\right] + \frac{1}{K_b}\left[J_T R_b^* - J_y \eta_b \frac{dR_b^*}{d\eta_b}\right] + \frac{1}{A^*}\frac{dV_b^*}{d\eta_b} = 0,$$

$$\left[J_T U_b^* - J_y \eta_b \frac{dU_b^*}{d\eta_b}\right] = \frac{\lambda}{A^{*2}K_b}\left\{\frac{d}{d\eta_b}\left(H_b^{*\omega}\frac{dU_b^*}{d\eta_b}\right)\right\},$$

$$\left[J_v V_b^* - J_y \eta_b \frac{dV_b^*}{d\eta_b}\right] - \frac{\lambda}{A^{*2}K_b}\left\{\frac{4}{3}\frac{d}{d\eta_b}\left(H_b^{*\omega}\frac{dV_b^*}{d\eta_b}\right)\right\}$$

$$+ \frac{\lambda}{A^* K_b}\left\{\frac{2}{3}\frac{d}{d\eta_b}\left(H_b^{*\omega}\left[J_T U_b^* - J_y \eta_b \frac{dU_b^*}{d\eta_b}\right]\right)\right.$$

$$\left. - \left[J_v\left(H_b^{*\omega}\frac{dU_b^*}{d\eta_b}\right) - J_y \eta_b \frac{d}{d\eta_b}\left(H_b^{*\omega}\frac{dU_b^*}{d\eta_b}\right)\right]\right\} = 0,$$

$$\left[J_T H_b^* - J_y \eta_b \frac{dH_b^*}{d\eta_b}\right] = \frac{\lambda}{A^{*2}K_b}\left\{\frac{1}{\sigma}\frac{d}{d\eta_b}\left(H_b^{*\omega}\frac{dH_b^*}{d\eta_b}\right)\right\}, \qquad (9.06)$$

neglecting terms proportional to $\xi_b^{1/(1+\omega)}$. The corresponding boundary conditions at the outer $(\eta_b = 1)$(derivable from Eq. (9.01)) are

$$U^*_{b,E} = V^*_{b,E} = R^*_{b,E} = H^*_{b,E} = 0 \,. \tag{9.07}$$

Within this framework, the boundary conditions at the plate ($\eta_b = 0$), with respect to tangential velocity-slip and no normal flow, become

$$1 - \frac{\lambda C_1}{A^* K_b^{3/2}} \left\{ H_b^{*(2\omega-1)/2} \frac{dU_b^*}{d\eta_b} \right\}_{,0} = V^*_{b,0} = 0 \,. \tag{9.08}$$

As far as the temperature-jump condition at the plate is concerned, there are two such conditions that render the equations of motion and all their boundary conditions locally self-similar as $\xi_b \rightarrow 0$. The first is that of a 'hot wall' with $T_{b,w} \rightarrow T^*_{b,w} =$ const. as $\xi_b \rightarrow 0$. For this 'hot wall' case, the temperature-jump condition becomes

$$T^*_{b,w} + \frac{\lambda C_2}{A^* K_b^{3/2}} \left\{ H_b^{*(2\omega-1)/2} \frac{dH_b^*}{d\eta_b} \right\}_{,0} = 0 \,. \tag{9.09a}$$

The second is that of an 'adiabatic wall' with $T_{b,w} \rightarrow (\text{const.}) \xi_b^{1/(1+\omega)} = \xi_b^{1/(1+\omega)} H^*_{b,w} = \xi_b^{1/(1+\omega)} H^*_{b,0}$ as $\xi_b \rightarrow 0$. For the 'adiabatic wall' case, the temperature-jump condition is

$$\left(\frac{dH_b^*}{d\eta_b} \right)_{,0} = 0 \,. \tag{9.09b}$$

Thus, Eqs.(9.06)-(9.09) represent the 'starting' ordinary differential equations and their boundary conditions. Their solutions provide the 'initial conditions' for the parabolic system of partial differential equations that defines the continuum, thin layer model that has been postulated.

References

[1] Bush, W. B., J. Fluid Mech. 25, 51 (1966).

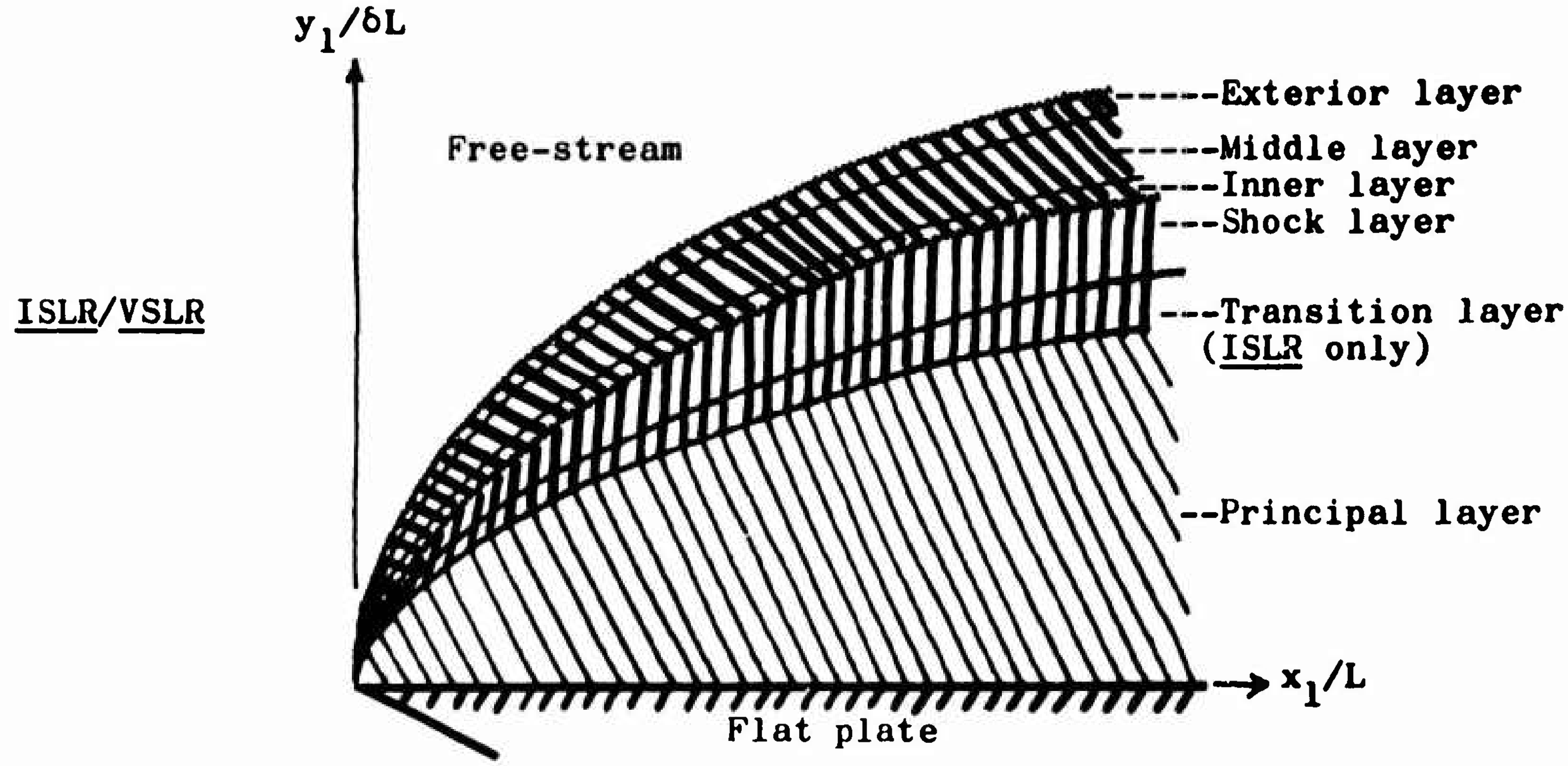
[2] Pan, Y. S. and Probstein, R.F., 3rd Intern'l Rarefied Gas Dyn. Symp'm., vol.II, 194 (Academic Press, New York, 1963).

[3] Pan, Y. S. and Probstein, R. F., Mass. Inst. of Tech. Fluid Mech. Lab. Public. no. 64-8 (1964).

[4] Garvine, R. W., AIAA J. 2, 1660 (1964).

[5] Garvine, R. W., Princeton Univ. Dept. of Aero. Eng'g. Rept. no. 755 (1965).

[6] Cole, J.D., J. Aero. Sci. 24, 448 (1957).
[7] Bush, W. B., J. Fluid Mech. 20, 333 (1964).



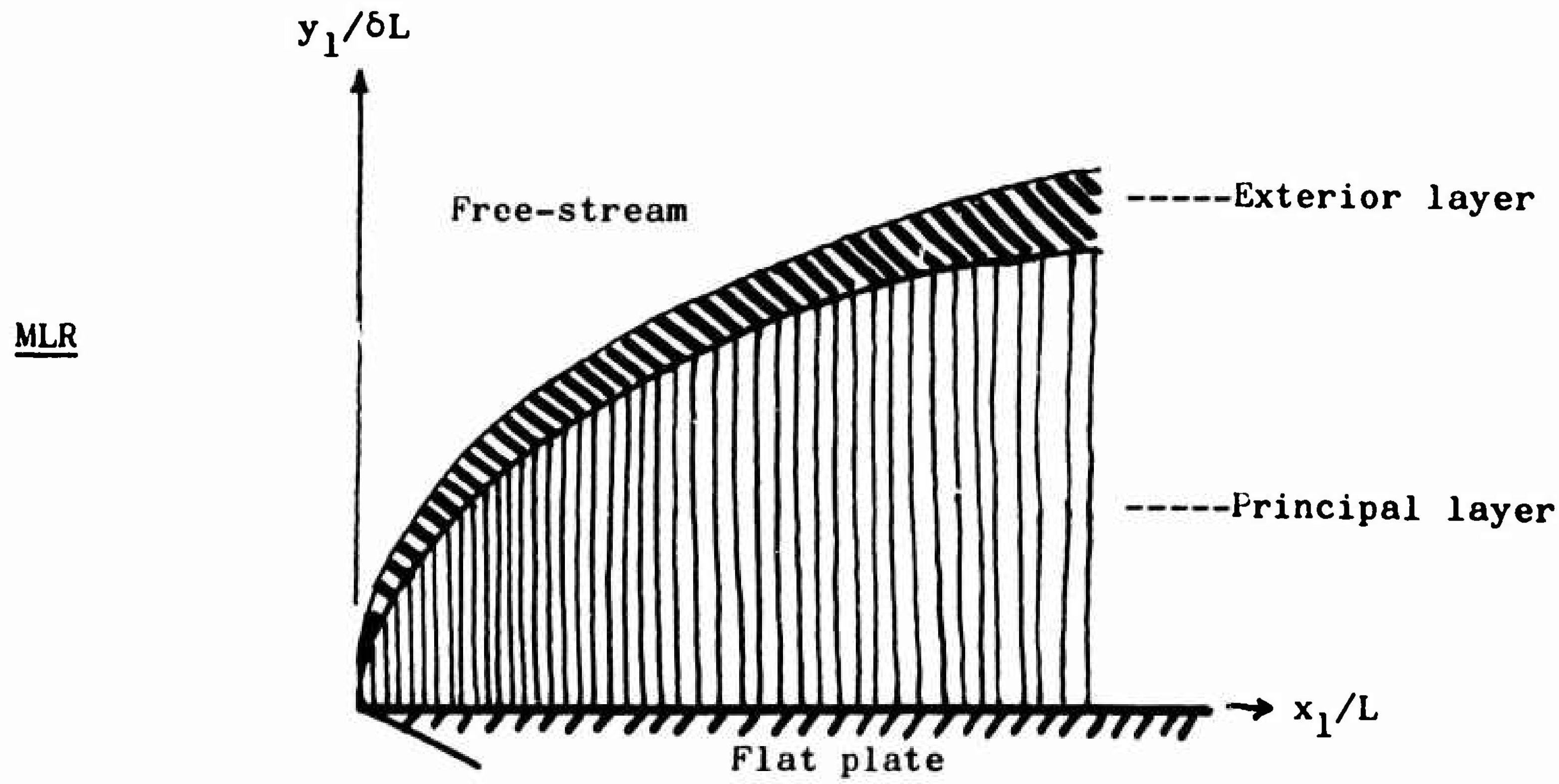


Fig. 1. Schematic Diagram of the Newtonian Hypersonic Strong-Interaction Regimes for Flow Past a Flat Plate.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Southern California Department of Aerospace Engineering Los Angeles, California 90007	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

ON THE NEWTONIAN HYPERSONIC STRONG-INTERACTION THEORY FOR FLOW PAST A FLAT PLATE

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Scientific Interim

5. AUTHOR(S) (First name, middle initial, last name)

BUSH, William B.

6. REPORT DATE September 1966	7a. TOTAL NO. OF PAGES Sixteen (16)	7b. NO. OF REFS Seven (7)
8a. CONTRACT OR GRANT NO. AF-AFOSR-697-66 b. PROJECT NO. c. 61445014 d. 681307	9a. ORIGINATOR'S REPORT NUMBER(S) USCAE 102 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) AFOSR 66-2403	

10. DISTRIBUTION STATEMENT

1. Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES PROCEEDINGS	12. SPONSORING MILITARY ACTIVITY Air Force Office of Scientific Research (SREM) 1400 Wilson Boulevard Arlington, Virginia 22209

13. ABSTRACT

The viscous hypersonic flow past the leading edge of a sharp flat plate, whose surface is parallel to an oncoming uniform flow, is analysed on the basis of a continuum model consisting of the Navier-Stokes equations and the velocity-slip and temperature-jump wall boundary conditions. It is assumed that the model fluid is a perfect gas having constant specific heats, a constant Prandtl number, σ, whose numerical value is order unity, and a normal viscosity coefficient varying as a power, ω, of the absolute temperature. Limiting forms of the solutions for such a flow are studied as: (1) the free-stream Mach number, M, goes to infinity; (2) the free-stream Reynolds number based upon the distance from the leading edge, R_L, goes to infinity; and (3) the 'Newtonian parameter', $\epsilon = (\gamma-1)/(\gamma+1)$, where γ is the ratio of the specific heats, goes to zero; such that the Newtonian hypersonic interaction parameter, $\chi_\epsilon = (\epsilon^{3+\omega} M^{2(2+\omega)}/R_L)^{1/2}$, goes to infinity.

Through the use of asymptotic expansions and matching, it is shown that, for $\omega < 1$, near the leading edge, the interaction of the thin (thickness ratio, δ, going to zero), viscous, high temperature, principal layer adjacent to the plate and the external flow produces three distinct strong-interaction $(N = 1/\epsilon M^2\delta^2 \ll 1)$ (sub-) regimes, for which the quantity δ is of order $\epsilon^m (m > 1/(1+\omega))$, $\epsilon^{1/(1+\omega)}$, and $\epsilon^{1/2}$, respectively, as the various limits are approached.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Rarefied Gas Dynamics Viscous Hypersonic Aerodynamics Leading Edge, Flat Plate						